AB Calculus   Name_____

Product Rule and Chain Rule Practice

Differentiate each function with respect to *x*.

1) $y = 4x^5(4x^4 - 2)$

2) $y = -2x^5(2x^2 - 3)$

3) $f(x) = (-x^3 - 1)(-x^5 - 1)$

4) $f(x) = (2x^2 + 2)(5x^4 + 4)$

5) $y = (-1 + x^{-4})(-x^5 - 1)$

6) $f(x) = \left(-2x^{\frac{5}{4}} - 1\right)(4x^5 + 1)$

7) $f(x) = \left(\sqrt[5]{x^2} + 4\right)(2x^2 + 4)$

8) $y = \left(-4x^{\frac{2}{3}} + 1\right)(3x^5 + 5)$

9) $f(x) = (4x^4 + 3)^2$

10) $f(x) = (-x^5 - 2)^4$

11) $y = ((-5x^4 - 2)^5 - 3)^4$

12) $y = ((-4x + 1)^5 + 3)^4$

13) $y = ((3x^5 - 5)^5 - 1)^4$

14) $y = ((-4x + 5)^2 + 1)^4$

Answers to Product Rule and Chain Rule Practice

1) $\dfrac{dy}{dx} = 4x^5 \cdot 16x^3 + (4x^4 - 2) \cdot 20x^4$

   $= 144x^8 - 40x^4$

2) $\dfrac{dy}{dx} = -2x^5 \cdot 4x + (2x^2 - 3) \cdot -10x^4$

   $= -28x^6 + 30x^4$

3) $f'(x) = (-x^3 - 1) \cdot -5x^4 + (-x^5 - 1) \cdot -3x^2$

   $= 8x^7 + 5x^4 + 3x^2$

4) $f'(x) = (2x^2 + 2) \cdot 20x^3 + (5x^4 + 4) \cdot 4x$

   $= 60x^5 + 40x^3 + 16x$

5) $\dfrac{dy}{dx} = (-1 + x^{-4}) \cdot -5x^4 + (-x^5 - 1) \cdot -4x^{-5}$

   $= 5x^4 - 1 + \dfrac{4}{x^5}$

6) $f'(x) = \left(-2x^{\frac{5}{4}} - 1\right) \cdot 20x^4 + (4x^5 + 1) \cdot -\dfrac{5}{2}x^{\frac{1}{4}}$

   $= -50x^{\frac{21}{4}} - 20x^4 - \dfrac{5x^{\frac{1}{4}}}{2}$

7) $f'(x) = \left(x^{\frac{2}{5}} + 4\right) \cdot 4x + (2x^2 + 4) \cdot \dfrac{2}{5}x^{-\frac{3}{5}}$

   $= \dfrac{24x^{\frac{7}{5}}}{5} + 16x + \dfrac{8}{5x^{\frac{3}{5}}}$

8) $\dfrac{dy}{dx} = \left(-4x^{\frac{2}{3}} + 1\right) \cdot 15x^4 + (3x^5 + 5) \cdot -\dfrac{8}{3}x^{-\frac{1}{3}}$

   $= -68x^{\frac{14}{3}} + 15x^4 - \dfrac{40}{3x^{\frac{1}{3}}}$

9) $f'(x) = 2(4x^4 + 3) \cdot 16x^3$

   $= 32x^3(4x^4 + 3)$

10) $f'(x) = 4(-x^5 - 2)^3 \cdot -5x^4$

    $= -20x^4(-x^5 - 2)^3$

11) $\dfrac{dy}{dx} = 4\left((-5x^4 - 2)^5 - 3\right)^3 \cdot 5(-5x^4 - 2)^4 \cdot -20x^3$

    $= -400x^3\left((-5x^4 - 2)^5 - 3\right)^3 \cdot (-5x^4 - 2)^4$

12) $\dfrac{dy}{dx} = 4\left((-4x + 1)^5 + 3\right)^3 \cdot 5(-4x + 1)^4 \cdot -4$

    $= -80\left((-4x + 1)^5 + 3\right)^3 \cdot (-4x + 1)^4$

13) $\dfrac{dy}{dx} = 4\left((3x^5 - 5)^5 - 1\right)^3 \cdot 5(3x^5 - 5)^4 \cdot 15x^4$

    $= 300x^4\left((3x^5 - 5)^5 - 1\right)^3 \cdot (3x^5 - 5)^4$

14) $\dfrac{dy}{dx} = 4\left((-4x + 5)^2 + 1\right)^3 \cdot 2(-4x + 5) \cdot -4$

    $= -32\left((-4x + 5)^2 + 1\right)^3(-4x + 5)$